AIAA 82-4272

Load Distribution on Deformed Wings in Supersonic Flow

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Arbitrary deformations which include the special cases of wing camber and twist provide unique problems to the analyst when localized loadings are to be determined. Other than twist and camber, deformations in the lifting surfaces may occur from high loadings due to severe maneuverability requirements or from aerodynamic heating or combinations thereof. Existing supersonic potential flow theories appear to be inadequate in predicting pressures under these conditions and numerical finite-difference methods have excessive computer requirements if the whole three-dimensional wing is modeled. In the present work a new method is presented which retains the simplicity of three-dimensional potential flow theories yet incorporates desirable features of finite-element techniques. The method utilizes planar three-dimensional finite wing theory overlaid with vorticity paneling to account for perturbations due to the deformations. The solution is stable (nonoscillatory) and requires minimal computer time and storage. Results for three deformed mean lines for two separate planform geometries are presented with excellent agreement with experimental data.

Nomenclature

b = wing span
B_L = coefficient of vorticity paneling loading term
c = local chord (nondimensional)
c_l = sectional lift coefficient
c_m = sectional moment coefficient
$\bar{c}$ = mean aerodynamic chord
C_L = total lift coefficient
C_{M_y} = total moment coefficient about the y axis
CR = root chord
CT = tip chord
ΔC_p = pressure loading coefficient $= C_{p_L} - C_{p_u}$
P = pressure loading parameter, "exact theory"
S = total wing area
S_0 = area of integration (forecone)
w = nondimensional downwash
x,y,z = Cartesian coordinates (field points)
α = angle of attack
β = Mach flow parameter $= \sqrt{M_\infty^2 - 1}$
γ = vorticity strength per unit area
η = nondimensional spanwise coordinate $= y/(b/2)$
θ = local deformation angle
θ_c = local deformation angle at a control point
ξ = nondimensional chordwise coordinate $= (x - x_{LE})/c$
ϕ = nondimensional velocity potential

Introduction

THE compressible flow form of the supersonic potential equation adequately describes the flow over most thin wings at supersonic Mach numbers. The problem is to find solutions which match the physical flow conditions, have some degree of numerical stability, and accurately predict the load distribution on the lifting surface. Most formulations currently in use were originally derived for thin planar wings and are quite adequate if the wing loading is relatively low. But because of the high maneuverability of certain missiles and aircraft, wing loadings may be created which tend to distort the mean camber surface, resulting in somewhat arbitrary deformations. The exact nature of the deformations are, of course, dependent on the aeroelastic properties of lifting surface. Prediction of localized loading under these conditions overextends present planar theories and thus these theories are inadequate in predicting local pressure loading even though basic potential flow conditions may still exist.

Received Aug. 24, 1981; revision received Feb. 2, 1982.

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Potential flow techniques ranging from the source solutions of Evvard[1] and Jones[2] to the constant pressure paneling technique of Carlson[3] have been successful in predicting the localized loading on thin planar wings with camber, but these methods have not been applied to arbitrary deformations.

At this point one may resort to numerical solutions of the nonlinear governing equations. Although numerical methods for solving the full nonlinear potential equations (or in some cases, the Navier- Stokes equations) provide the most accurate solutions (e.g., Ref. 4), they have excessive, computer requirements, are very expansive, and have limited applicability. Potential flow solutions such as "three-dimensional supersonic flow theory" formulated by Evvard,[1] Harmon et al.,[5] and Malvestuto et al.[6] provide quick accurate solutions for isolated planar wings with straight leading and trailing edges, but are not generally applicable to the deformed wing problem. Other linearized theories developed to treat camber and twist, such as the supersonic Kernel function method, have been used with varying degrees of success.

Some analysts, such as Cunningham[7] and Curtis and Lingard,[8] used assumed functions to solve the wing problem in a similar manner to that employed in subsonic Kernel function methods by Purvis.[9] Others, such as Miranda[10] and Carlson et al.,[3,11,12] solve the Kernel integral in a manner which utilizes a discrete element approach. The most serious disadvantage of the Kernel function method is that planforms with discontinuous pressure distribution are very difficult to treat. Since the assumed pressure functions are smooth, the convergence to a discontinuous distribution is very slow. When discrete elements are used, high-frequency disturbances propagate along Mach lines and may increase in amplitude. Cunningham[7] avoided the convergence problem by using exact theory weighting functions, which contain the appropriate discontinuities. In the latter case, Carlson et al.[3,11,12] employed a "terminal smoothing" procedure to eliminate the undesired wiggles.

The present method proposes to combine the assumed pressure function approach for solving the supersonic Kernel integral with the discrete element technique. Results will show that this approach retains the speed and accuracy advantages of the Kernel function method without encountering any of the previously mentioned oscillatory problems provided that the deformations remain in the perturbation category and do not dominate the solution. Even though the present theory is a potential flow solution, it is applicable to arbitrary moderate wing deformations. The solution technique is fundamentally based on three-dimensional supersonic flow theory as outlined in Refs. 5, 6, and 13, but is modified to account for wing deformations. The deformed wing is then "overlaid" with a vorticity paneling sheet whose strength everywhere is determined through matrix inversion techniques and the vorticity solution is subsequently added to the basic three-dimensional supersonic solution. This is accomplished by subdividing the wing planform into very small panels over which the pressure is assumed constant. Integration is completed over each subpanel in closed form and results may then be written in summation notation. The vorticity paneling produces perturbation velocities which account for deformations in the wing surface.

Theory

Fundamental Equations

The theoretical approach is a potential flow formulation in which solutions derived from three-dimensional supersonic theory and vorticity paneling are added to form the general solution of the supersonic potential equation.

$$\phi(x,y,z) = \frac{1}{2\pi}\iint_{S_0} \frac{\gamma(x_0,y_0)}{(y-y_0)^2+z^2}z \times\left[\frac{(x-x_0)}{\sqrt{(x-x_0)^2-\beta(y-y_0)^2-\beta^2z^2}}\right]\mathrm{d}x_0\mathrm{d}y_0 \quad (1)$$

Just as in subsonic flow, the vorticity distribution $\gamma(x_0,y_0)$ may be replaced by the pressure loading coefficient $\frac{1}{2}\Delta C_p(x_0,y_0)$. The downwash equation is obtained by differentiation in the standard manner and when evaluated in the $z=0$ plane (planar flow) becomes

$$w(x,y) = \frac{1}{4\pi}\iint_{S_0} \frac{\Delta C_p(x_0,y_0)}{(y-y_0)^2} \times\left[\frac{(x-x_0)}{\sqrt{(x-x_0)^2-\beta^2(y-y_0)^2}}\right]\mathrm{d}x_0\mathrm{d}y_0 \quad (2)$$

where the area S_0 is that contained on the wing surface in the upstream running Mach cone emanating from the field point (x,y).

Although the vorticity paneling solution which satisfies Eq. (1) is formulated for planar flow, it may be used to account for nonplanar velocity components if the slope of the nonplanar surface is properly taken into account. The downwash that a lifting surface in supersonic flow induces on itself is unique in that the downwash point is located on the boundary of the region of integration. The integral gives the appearance of being improper due to the singularity at $y=y_0$ within the region of integration. This integrand is the limiting form, in the $z=0$ plane, of a more general nonsingular integrand. Consequently, the integral would be expected to yield a finite result for the downwash just as in the subsonic case. There is an upwash singularity along Mach waves through (x_0,y_0) due to the square root term in the denominator. As noted in Ref. 3: "... this same upwash field, with the large values of upwash near the Mach cone limits is also largely responsible for the difficulties in representing supersonic-flow phenomena by means of finite-element techniques." This problem does not appear in the current method, which contains finite-element-type terms, and, in fact, the influence of a discrete panel along its Mach waves will be shown to be a finite value. Equation (2) is the starting point adopted by most authors[3,7] for the planar wing case.

The first step in solving the integral equation is to determine the form of the unknown pressure loading ΔC_p, which unfortunately appears under the integral sign. As was demonstrated for subsonic flow,[9] exact theoretical results, in this case from three-dimensional supersonic theory, will be used as a weighting function and will be multiplied by an assumed functional distribution with unknown coefficients. The form of the weighting function is particularly important in supersonic flow since the pressure distribution has sharp breaks (discontinuities in slope) along certain Mach lines. This procedure has been used successfully by Cunningham.[7]

The assumed pressure loading consists of the basic three-dimensional supersonic flow term plus the perturbation term from the vorticity paneling and is written as

$$\Delta C_p(x_0,y_0) = P(x_0,y_0)\sin(\alpha+\theta_0) + P(x_0,y_0)\sum_{n=0}^{N}\sum_{m=0}^{M} B_L\eta^m\cos(n\pi\xi/2) \quad (3)$$

The first expression on the right-hand side is the three-dimensional loading term modified to account for local wing deformations and is valid for both subsonic and supersonic leading edges. The second term uses the three-dimensional pressure functions $P(x_0,y_0)$[5,6,13] as a weighting function which, when multiplied by the unknown coefficient B_L, and the series $\eta^m\cos(n\pi\xi/2)$ accounts for small deformations in the wing surface in the upstream Mach cone from the field point (x,y). The order of the terms in the second expression seldom exceeds 3 or 4 and for planar wings, the entire second term is of course unnecessary. Increasing the number of terms in the second expression to more than 12 produces a little better agreement with experimental data but an excessive number of terms actually degrades accuracy. The reason for this is that the double summation term is actually a correction to the planar solution and thus describes the perturbations due to deformations in the wing. The perturbations are small and thus the number of correction terms should not be excessive or else the matrix to be inverted for the unknown coefficients begins to look somewhat singular. In the present work, satisfactory results were obtained for correction terms ranging in number from 9 to 49 with 20 to 25 producing the better agreement with experimental data. The form of the correction terms, η^m (polynomial) in the spanwise direction and the truncated Fourier-type terms in the chordwise direction were assumed based somewhat on the polynomial assumptions of Cunningham[7] but primarily on previous success with these types of terms in subsonic Kernel function solutions.[9,13] The number of terms in the spanwise or chordwise direction were not examined independently; however, because of the nature of the power series terms, very large values of m will lead to an ill-conditioned matrix.

The weighting function, $P(x_0,y_0)$, has several forms depending on whether the wing loading and trailing edges are subsonic or supersonic. The functional formulation for $P(x_0,y_0)$ is presented in Refs. 5, 6, and 13.

With the desired form of the pressure function determined, the next task is to evaluate the downwash integral. The evaluation does not proceed in the classical manner of treating Kernel functions (i.e., numerical quadrature), even though functional pressure loading distributions from discrete element approaches (see, for example, Woodward[14]) is used to simplify the integration procedure.

First, the wing surface is subdivided into a large number of small finite panels or elements. Equation (2) for the downwash may then be written as

$$w(x,y) = \sum\sum_{S_0}\Delta w(x,y) \quad (4)$$

where $\Delta w(y,y)$ denotes the downwash at (x,y) due to an element or portion of an element in the Mach forecone from (x,y). By judiciously choosing the manner in which the wing is subdivided, each element or portion thereof lying in the region of integration S_0 will be either a rectangle or triangle, as shown in Fig. 1. The integral expression for a typical element is

$$\Delta w(x,y) = \frac{1}{4\pi}\int_{y_1}^{y_2}\int_{x_1}^{x_2} \frac{\Delta C_p(x_0,y_0)}{(y-y_0)^2} \times \left[\frac{(x-x_0)}{\sqrt{(x-x_0)^2-\beta^2(y-y_0)^2}}\right]\mathrm{d}x_0\mathrm{d}y_0 \tag{5}$$

Note that for sufficiently small elements the pressure loading is essentially constant over the entire element. Consequently, $\Delta C_p(x_0,y_0)$ may be taken outside the integral and evaluated at the element centroid $(\bar{x}_0,\bar{y}_0)$.

Purvis[13] has shown that the resulting integral with $\Delta C_p(x_0,y_0)$ taken outside may be evaluated in closed form when the appropriate limits of integration are known. For rectangular elements, such as element A in Fig. 1, the result is

$$\Delta w(x,y) = \frac{\beta\Delta C_p(\bar{x}_0,\bar{y}_0)}{4\pi}[-K(x_2,y_2)+K(x_2,y_1)+K(x_1,y_2)-K(x_1,y_1)] \tag{6}$$

where

$$K(x,y;x_i,y_j) = \frac{\sqrt{(x-x_i)^2-\beta^2(y-y_j)^2}}{\beta(y-y_j)} + \arctan\frac{\beta(y-y_j)}{\sqrt{(x-x_i)^2-\beta^2(y-y_j)}} \tag{7}$$

For triangular elements bounded by Mach lines, such as elements B and C, the lower x limit is a function of y_0. The form of this function causes certain terms to vanish during integration. Results for both elements B and C are then

$$\Delta w(x,y) = \frac{\beta\Delta C_p(\bar{x}_0,\bar{y}_0)}{4\pi}[K(x_1,y_2)-K(x_1,y_1)] \tag{8}$$

where the K function is given by Eq. (7). For element D, which includes the point (x,y) on its boundary, two separate integrals as well as the poles of two inverse-tangent functions must be evaluated. The integration and evaluation yields the well-known Ackeret results[15] for an infinite span wing:

$$\Delta w(x,y) = -\beta\Delta C_p(\bar{x}_0,\bar{y}_0)/4 \tag{9}$$

Note that for element D, the value of the K function is $-\pi$.

The exact integrals for elements cut by the wing leading edge, such as E and F, yield three complicated expressions, depending on whether the leading edge is subsonic, sonic, or supersonic. Rather than using the more complicated results, the downwash due to these elements is approximated by using one-half of the value due to a rectangular element with the same x and y dimensions. The outlines of such elements are shown by dashed lines in Fig. 1. The equations for the downwash is then one-half the value as for the rectangular elements. The pressure loading $\Delta C_p(\bar{x}_0,\bar{y}_0)$ is evaluated at the centroid of the leading edge triangular element. This approximation would be expected to have little effect except for control points close to the leading edge of very highly swept wings.

The only remaining task is to evaluate the unknown coefficient B_L in the pressure loading Eq. (3). This requires introducing the tangency condition

$$\sin(\alpha+\theta_c)_{x,y} + w(x,y) = 0 \tag{10}$$

where $\sin(\alpha+\theta_c)_{x,y}$ is the nondimensional freestream velocity component normal to the deformed wing at (x,y) and θ_c is the local deformation angle at a control point. Rearranging and combining Eqs. (3-5) and (10) yields

$$\sum_{i=1}^{I}(\Delta K)_i\left[P(\bar{x}_0,\bar{y}_0)\sum_{n=0}^{N}\sum_{m=0}^{M}B_L\eta^m\cos\frac{n\pi\xi}{2}\right]_i = \frac{4\pi}{\beta}\sin(\alpha+\theta_c) + \sum_{i=1}^{I}(\Delta K)_i[P(\bar{x}_0,\bar{y}_0)\sin(\alpha+\theta_0)]_i \tag{11}$$

where $L=n+mN$, and θ_0 is the local slope of each ith element.

The above equation represents a set of L simultaneous linear algebraic equations and is conveniently arranged in matrix form for a Gaussian reduction solution for the unknown coefficients B_L. Once the unknown coefficients are determined, the localized loading is obtained from Eq. (3). Although not highly significant to the solution (unlike subsonic results), the nondimensional location of the control points are approximately chosen using a cosine distribution as

$$XO_i = 0.5\left(1-\cos\frac{\pi i}{NC}\right) \tag{12}$$

and

$$YO_j = 0.5\left(1-\cos\frac{\pi i}{NS+1}\right) \qquad j=NC(i-1)+1 \tag{13}$$

where NC is the number of control points in the chordwise direction and NS is the number of control points in the spanwise direction. It should be noted that even though the cosine distribution is not a critical factor, control points should not be located close to the leading edge. After the points are approximately chosen using Eqs. (12) and (13), they are shifted slightly to the centroid of the subelement on which

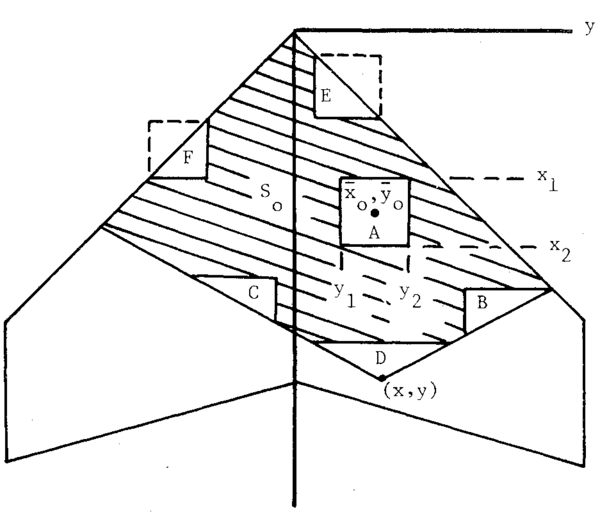

Fig. 1 Schematic of integration elements for a general wing planform.

they lie and the last row near the wing trailing edge is shifted to the trailing edge.

The sectional lift and pitching moment are defined and computed from

$$cc_l = c\int_{\xi=0}^{\xi=1} \Delta C_p \mathrm{d}\xi \tag{14}$$

and

$$(c^2 c_m)_y = -c^2 \int_{\xi=0}^{\xi=1} (\Delta C_p)\xi \mathrm{d}\xi - (x_{\mathrm{LE}}/c)(cc_l) \tag{15}$$

and the total lift and moment then become

$$C_L = (b/S)\int_{\eta=0}^{\eta=1} cc_l \mathrm{d}\eta \tag{16}$$

and

$$C_{M_y} = (b/S\bar{c})\int_{\eta=0}^{\eta=1} (c^2 c_m)_y \mathrm{d}\eta \tag{17}$$

Because of the nature of the terms involved, Eqs. (14-17) must be integrated numerically.

Comparison with Experimental Results

Comparison with experimental data of the theoretical analysis as previously presented is divided into two categories: planar wings and deformed wings. In many cases, the planform shapes chosen were dictated by available experimental data. In some cases, experimental data were available for both deformed and undeformed wings with the same planform shape.

Planar Wings

For planar wings, the theoretical solution for the pressure loading is basic three-dimensional supersonic theory as presented in Refs. 5 and 6; no vorticity paneling is required. The double summation terms in Eq. (3) are omitted and only the $P(x_0,y_0)\sin(\alpha)$ term is used to compute the pressure.

A summary of the planar results is presented in order to validate the computer program, and to provide a means of comparison with deformed wing results for similar planform shapes. A tabular summary for three wing configurations shown in Fig. 2 is presented in Table 1. In most cases, localized pressure loadings and the sectional coefficients agree quite well with experimental data. The comparison of total coefficients, as presented in Table 1, is indicative of the kind of agreement to be expected with the planar theory. It should be noted here that if vorticity paneling is overlaid on a planar wing, the computed strength is essentially zero; i.e., only small corrections are made to the three-dimensional planar results.

Deformed Wings

These same wings under various kinds of deformation were also analyzed by including vorticity paneling as well as the three-dimensional theoretical loading terms in the ΔC_p distribution to account for perturbations produced by the wing deformations. Deformation shapes of three wings are shown in Fig. 3. The delta wing (No. 1) in Fig. 3 was cambered and the slope relative to the chord line of the mean camber line was determined to be

$$\theta = -3.00825 + \tan^{-1}[\tan 6 \deg (\xi - \xi\eta + \eta)] \tag{18}$$

where $\eta = y/(b/2)$ and $\xi = (x - x_{\mathrm{LE}})/c$. The camber (see Fig. 3) is uniform in the spanwise direction and produces a maximum deflection angle of 3 deg relative to the chord line. Although this seems like a small task, this is actually a severe test of the theory and its agreement with experimental data. In Figs. 4 and 5, sectional results are presented for two deformed configurations at Mach numbers of 2.01 and angle of attack of 6 deg.

For the cambered delta wing, agreement of the sectional coefficients is quite good over the entire span. It is noted that the inclusion of the doublet paneling terms in the mathematical formulation correctly accounts for the deviation from planar results.

For the same planform shape, linear spanwise twist is added to the camber deformation such that the mean camber surface is now defined as

$$\theta = \tan^{-1}[0.140541(\xi - \xi\eta + \eta)\eta] \tag{19}$$

Results of this configuration are shown in Fig. 5. Similar results are obtained for this case as was observed for the cambered (alone) case. Agreement is better at the inboard stations than at the outboard stations.

The next wing subjected to the theoretical analysis was a "warped" trapezoidal wing (see Fig. 3). For both $M = 1.61$ and 2.01, the leading edge is supersonic and multiple "fold-over" regions occur because of the low aspect ratio

Table 1 Comparison of theoretical lift and pitching moment coefficients with experimental data for three planar wings

		Delta wing, $M_\infty = 2.01$	Low Æ trapezoidal wing, $M_\infty = 2.01$	High Æ trapezoidal wing, $M_\infty = 1.61$
C_{L_α}	Experiment	0.029	0.033	0.048
$\alpha = 0$	Theory	0.030	0.033	0.056
$(C_{M_\alpha})_y$	Experiment	−0.029	−0.019	−0.064
$\alpha = 0$	Theory	−0.030	−0.019	−0.075

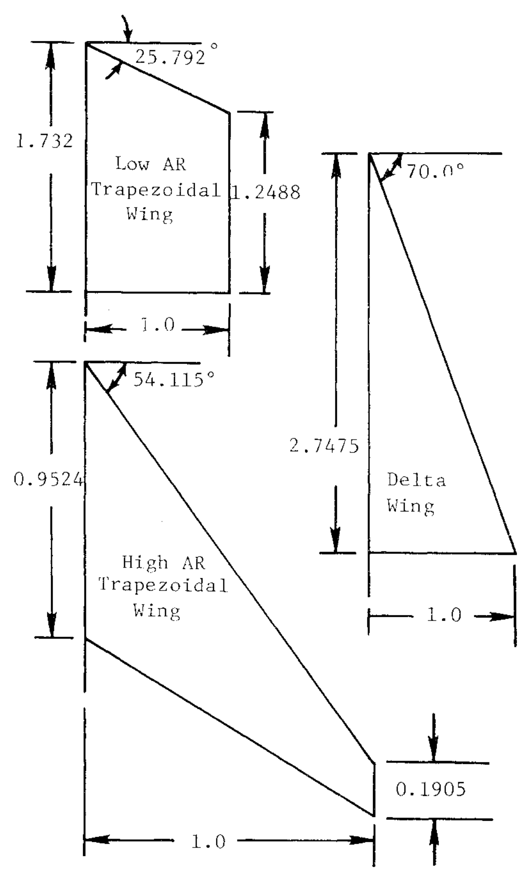

Fig. 2 Planform shapes for three planar wing configurations.

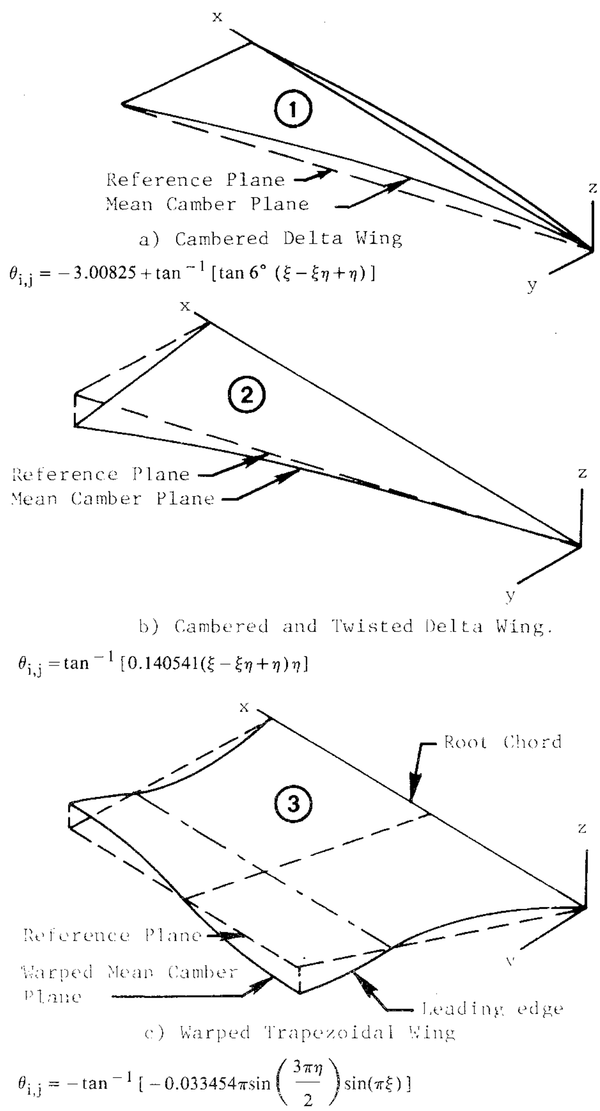

Fig. 3 Schematic of three deformed wing shapes.

trapezoidally shaped wing. The equation governing the mean warped camber surface is

$$\theta = -\tan^{-1}[-0.033454\pi\sin(3\pi\eta/2)\sin(\pi\xi)] \qquad (20)$$

Localized pressure loading results for this deformed configuration are presented in Fig. 6. The agreement over the entire wing is remarkably good. The vorticity paneling correctly accounts for deformations in the surface even at the wing tip where poorest agreement is usually expected. For other configurations, such as the delta planforms, agreement with experimental data is not always this good but is still significantly better than planar theory.

In Fig. 7, the sectional normal force and pitching moment are shown for the warped trapezoidal wing. As expected, the sectional normal force coefficients are predicted quite well with the theory as are the sectional moment coefficients. The total integrated lift and pitching moment coefficients are, of course, quite close to the measured experimental results.

Mach number effects are included in the potential equation and thus good agreement can be expected with experimental data for the applicable linear range. Figure 8 is a comparison between the present theory and experimental data for two different Mach numbers. Magnitude as well as correct trends are correctly predicted.

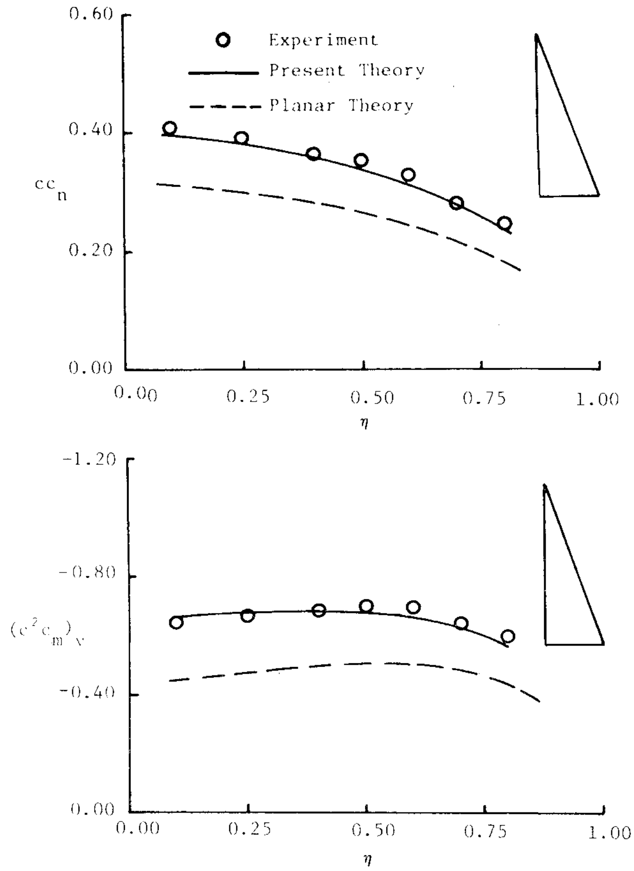

Fig. 4 Sectional normal force and pitching moment for a cambered delta wing; $M = 2.01$, $\alpha = 6.0$ deg.

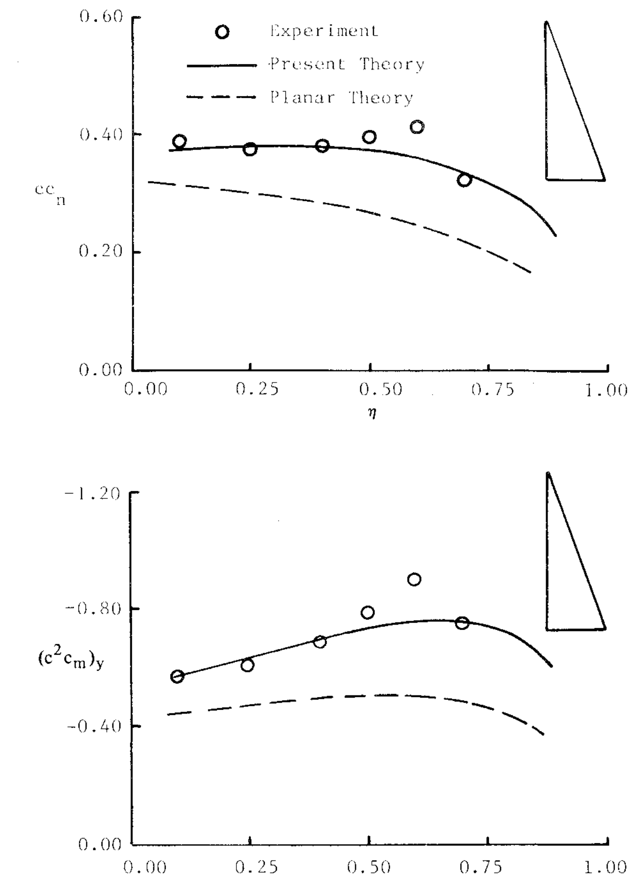

Fig. 5 Sectional normal force and pitching moment for a cambered and twisted delta wing; $M = 2.01$, $\alpha = 6.0$ deg.

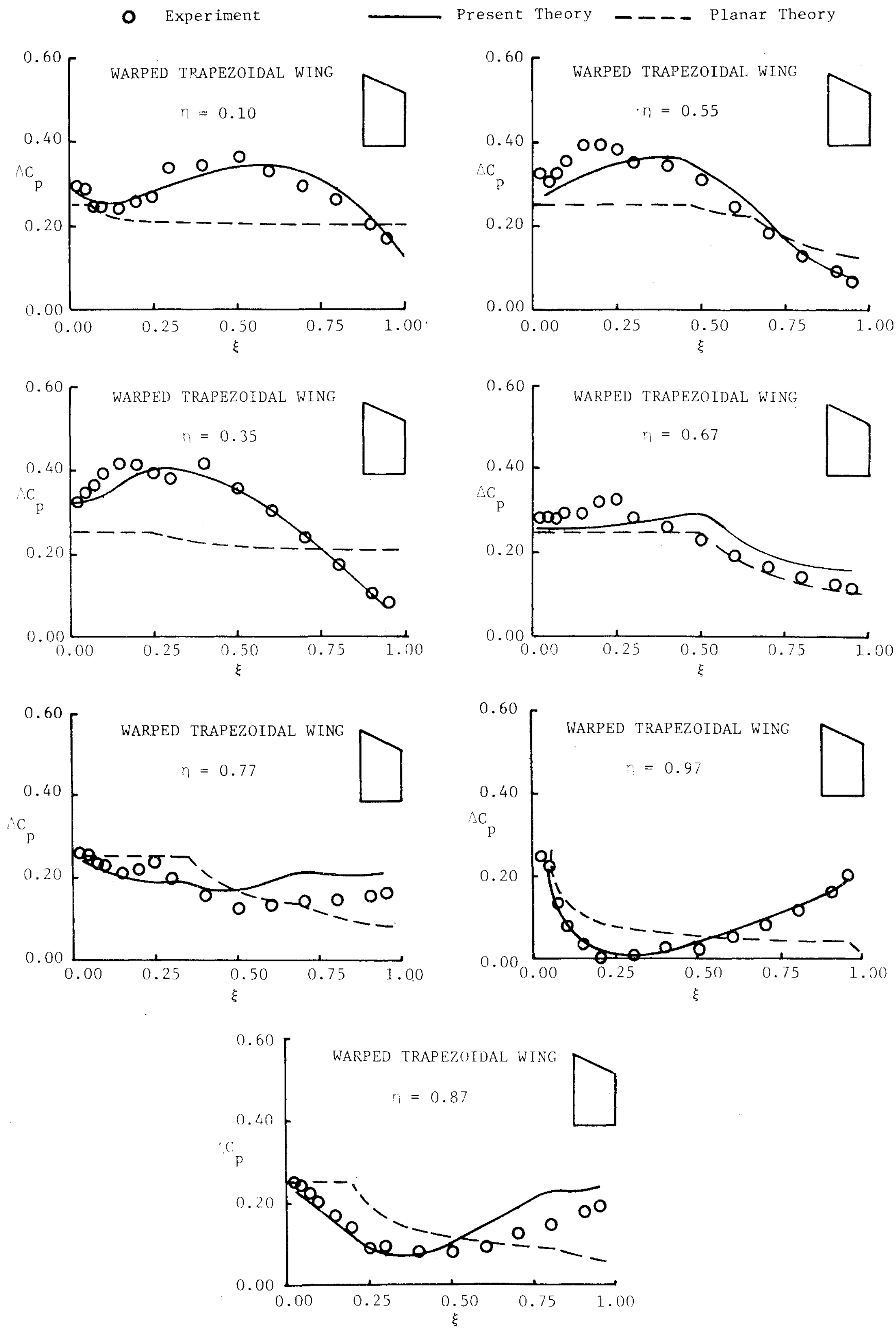

Fig. 6 Pressure distribution on a warped trapezoidal wing; $M = 2.01$, $\alpha = 6.0$ deg.

Concluding Remarks

A method for determining the localized pressure loading on a deformed wing in supersonic flow has been developed. It retains the ease and simplicity of planar three-dimensional wing theory yet captures the essential features of nonplanar numerical methods. Even though a (small) matrix solution is required for the vorticity paneling strength, there are no oscillatory or numerical stability problems. Correction terms ranging up to 49 in number have been successful in predicting localized pressures for deformed wings. Above this number, problems were encountered in inverting the 49×49 matrix and resulting pressures were unreliable, at least for the configurations investigated. Possible sources for the numerical difficulties for these large number of terms include 1) a

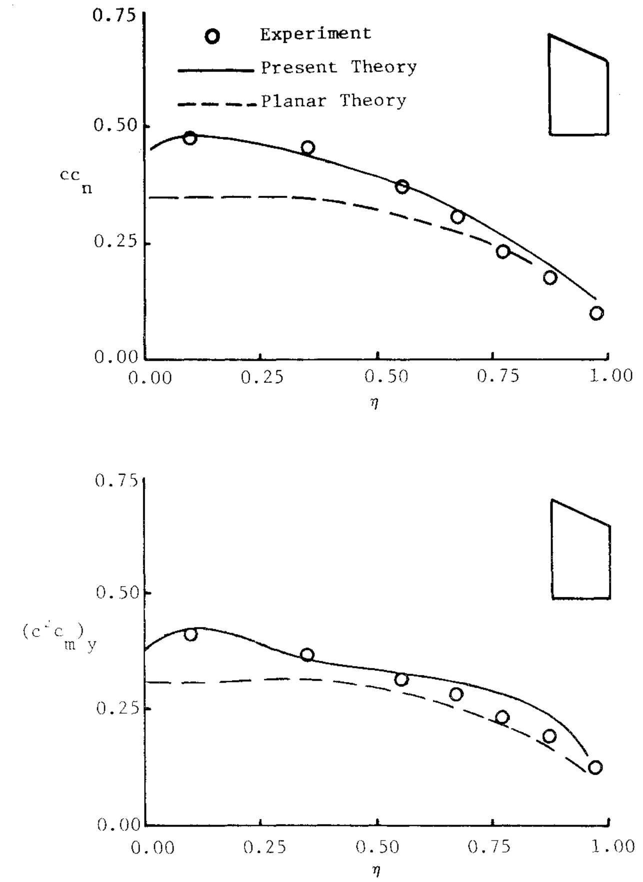

Fig. 7 Sectional normal force and pitching moment for a warped trapezoidal wing; $M = 2.01$, $\alpha = 6.0$ deg.

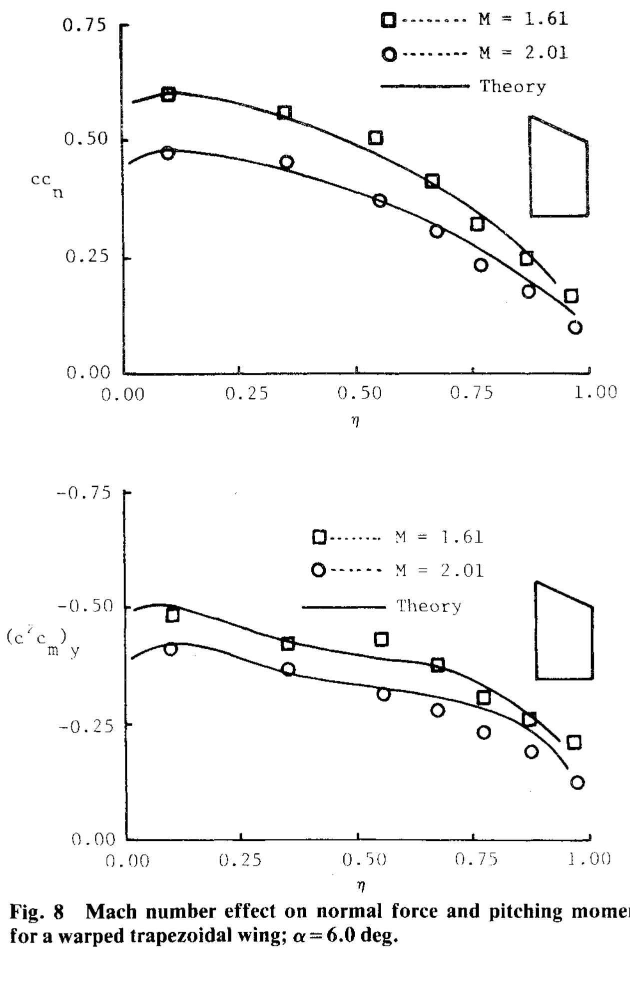

Fig. 8 Mach number effect on normal force and pitching moment for a warped trapezoidal wing; $\alpha = 6.0$ deg.

combination of round-off error in the (IBM) computer and use of power series correction terms in the spanwise direction, 2) control points located too close to the leading edge, and 3) "overkill," trying to use too many correction terms to correct something already nearly correct. This is analogous to fitting a linear curve with a 50th deg polynomial. It should be noted, at this point, that 20 correction terms are adequate at least for all the cases investigated. This conclusion is quite evident from agreement with experimental data.

The method is applicable to cambered wings, cambered and twisted wings, and any other arbitrary type of deformations as long as the flow does not separate. Planform shapes with subsonic or supersonic leading edges are easily handled and Mach number effects are properly accounted for. The method accurately predicts the localized pressure loadings and resulting integration produces excellent agreement of the sectional and total coefficients with experimental data.

Acknowledgment

This research was sponsored by the U.S. Army Research Office and the Army Missile Command under Contract DAAG29-78-G-0036. The assistance of personnel at the Army Missile Command and Army Research Office is greatly appreciated.

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